

AZIMUTHAL ANISOTROPY IN DILUTE DENCE LIMIT AT HIGH TRANSVERSE MOMENTUM

Vladimir Skokov



January 9, 2015

DISCLAIMER

- This talk is about anisotropy at high transverse momentum, $k_\perp > 3 \text{ GeV}$, corresponding to small resolution scale for conjugate variable $r_\perp < .07 \text{ fm}$.

Recent ATLAS results: anisotropy persists up to $k_\perp \approx 10 \text{ GeV}$.
Imperative to apply short-scale QCD dynamics.

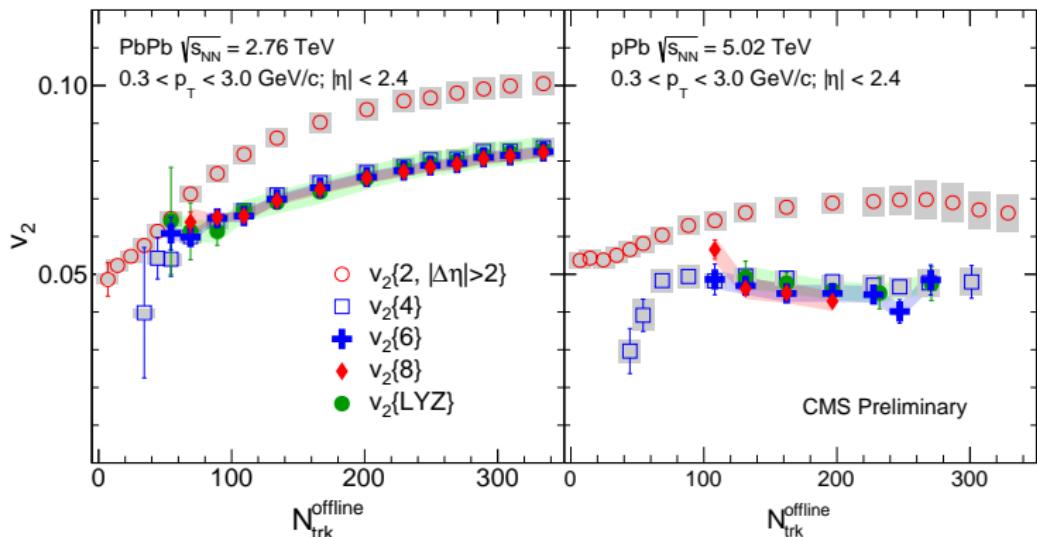
- Microscopic approach in this talk: CGC.
Dilute-dense limit (relevant for pA collisions).
- Elliptic anisotropy only, some comments on odd harmonics
- Comparison to experiment: we are not there yet and without transparent microscopical computations there is not point to try to describe data. But for record: yes, it is possible.

OUTLINE

- Preliminaries: Experimental results for pA collisions at LHC
- Motivation: Azimuthal asymmetry from connected diagrams in dilute-dense limit and high order cumulants
- Single-particle azimuthal asymmetry from initial state
- Azimuthal anisotropy in MV model: numerical results
- JIMWLK evolution: fate of anisotropy
- Conclusions

EXPERIMENTAL RESULTS: PA I

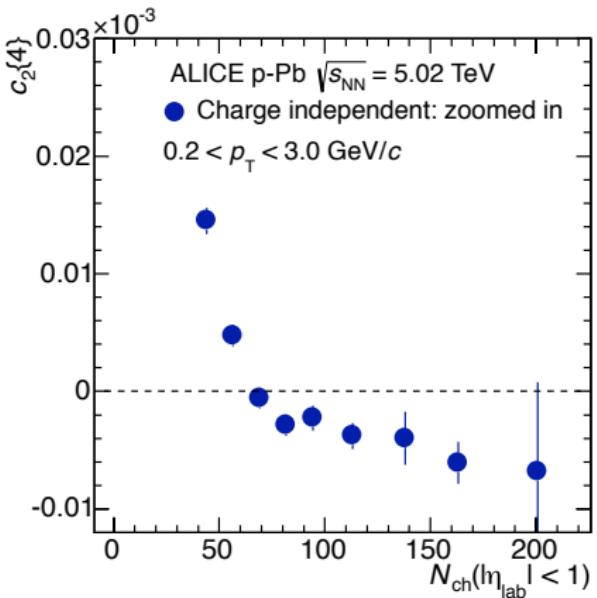
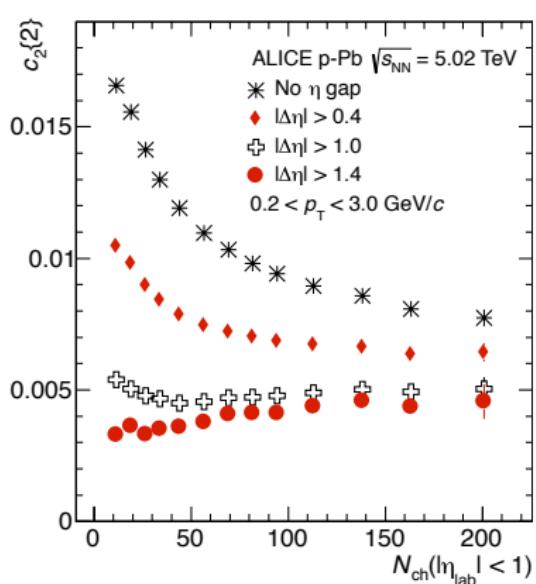
- Large azimuthal anisotropy in pA collisions at LHC
- Non-trivial multi-particle correlations. For two particles, correlation function has maxima at $\phi = \pi$ and $\phi = 0$.
- Fourier components and cumulants analysis (details later):



Hierarchy of harmonics $v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$ in high multiplicity events: • expected in AA collisions (“hydrodynamic flow”); • surprising for pA?!

EXPERIMENTAL RESULTS: PA II

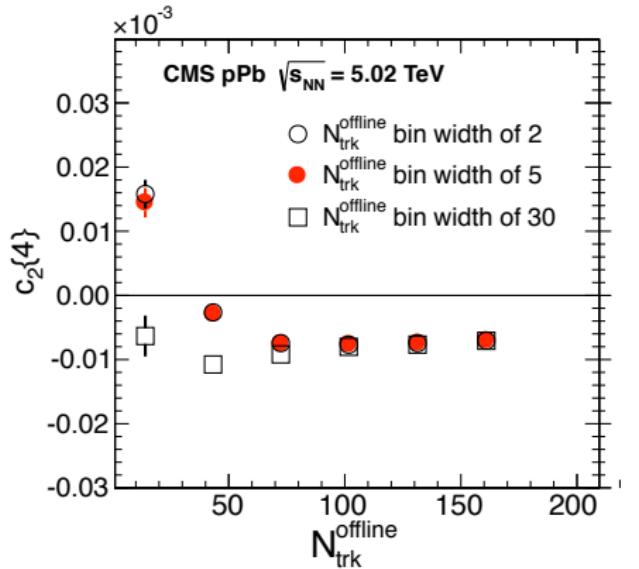
- Negative $c_2\{4\} \equiv -v_2^4\{4\}$ at high multiplicity
- Change of sign at $N \approx 60$.



Two distinct regimes defined by sign of $c_2\{4\}$

EXPERIMENTAL RESULTS: PA II

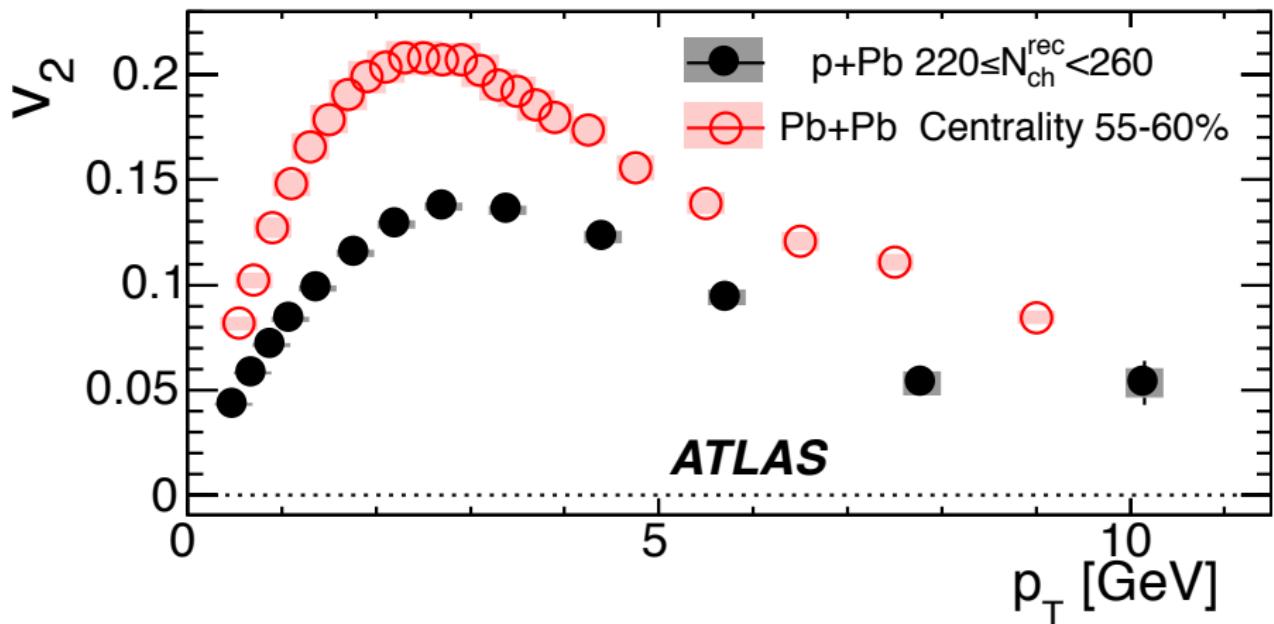
- Negative $c_2\{4\} \equiv -v_2^4\{4\}$ at high multiplicity
- Change of sign at $N \approx 60$.



Two distinct regimes defined by sign of $c_2\{4\}$

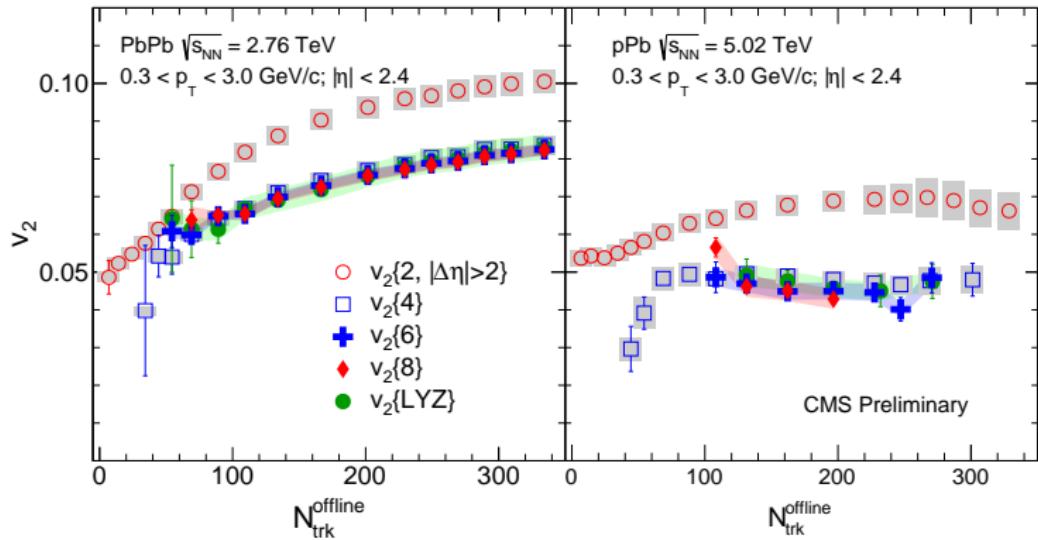
EXPERIMENTAL RESULTS: PA III

- Anisotropy persists to high momentum



HIERARCHY IN PA

Our original goal was to establish hierarchy in dilute-dense limit



SCATTERING CROSS SECTION FOR DILUTE-DENSE LIMIT I

- In eikonal approximation, parton propagation is described by light-like Wilson line

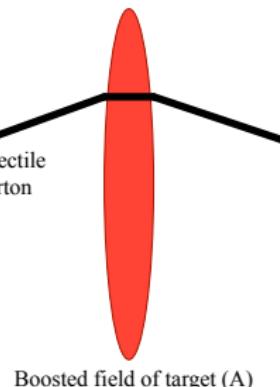
$$V(\vec{x}) = \mathcal{P} \exp \left(ig \int dx^- \textcolor{red}{A}^+(x^-, \vec{x}) \right)$$

- Scattering of quark off target in dipole model

$$\langle S_1 \rangle = \frac{1}{d_{\mathcal{R}}} \left\langle \text{tr}_{\mathcal{R}} V(\vec{x}_\perp) V^\dagger(\vec{y}_\perp) \right\rangle; \quad k_\perp \propto 1/r_\perp; \quad \vec{r}_\perp = \vec{x}_\perp - \vec{y}_\perp$$

$d_{\mathcal{R}}$ is dimension of representation \mathcal{R}

$$S_A(\vec{r}) = \frac{N_c^2 |S_F(\vec{r})|^2 - 1}{N_c^2 - 1}$$



$S_F(\vec{r})$ can be complex, while $S_A(\vec{r})$ is real.
(similar to Polyakov Loop)

SCATTERING CROSS SECTION FOR DILUTE-DENSE LIMIT II

- Scattering to high transverse momentum corresponds to small $|\vec{r}| \propto 1/k_{\perp}$. Gradient expansion of vector potential $\textcolor{red}{A}^+(x^-, \vec{x})$ gives (fundamental representation only)

$$\langle S_1(\vec{r}, \vec{b}) \rangle - 1 = \left\langle \frac{(ig)^2}{2N_c} \text{tr} \left(\vec{r} \cdot \vec{\textcolor{red}{E}}(\vec{b}) \right)^2 + \frac{1}{2} \left[\frac{(ig)^2}{2N_c} \text{tr} \left(\vec{r} \cdot \vec{\textcolor{red}{E}}(\vec{b}) \right)^2 \right]^2 + O(r^6) \right\rangle$$

Light-cone electric field of target in covariant gauge

$$\textcolor{red}{E}^i(\vec{b}) = \int dx^- F^{+i} = -\partial^i \int dx^- \textcolor{red}{A}^+(x^-, \vec{b}).$$

- For m -quarks (only leading order is shown)

$$\langle S_m \rangle - 1 = \left(\frac{(ig)^2}{2N_c} \right)^m \left\langle \text{tr}(\vec{r}_{1,\perp} \vec{\textcolor{red}{E}}_1)^2 \text{tr}(\vec{r}_{2,\perp} \vec{\textcolor{red}{E}}_2)^2 \cdots \text{tr}(\vec{r}_{m,\perp} \vec{\textcolor{red}{E}}_m)^2 \right\rangle; \quad \vec{\textcolor{red}{E}}_i = \vec{\textcolor{red}{E}}(\vec{b}_{i,\perp})$$

- By knowing $\langle \vec{\textcolor{red}{E}}(\vec{b}_1) \vec{\textcolor{red}{E}}(\vec{b}_2) \rangle$, one can compute S_m , cumulants, $c_n\{m\}$ and harmonics $v_n\{m\}$ of azimuthal anisotropy.

SCATTERING CROSS SECTION FOR DILUTE-DENSE LIMIT III

- A Event averaging corresponds to averaging over target ensemble, which is defined by field-field correlator $\langle \vec{E}(\vec{b}_1) \vec{E}(\vec{b}_2) \rangle$. Conventionally, in McLerran-Venugopalan model one uses

$$\frac{g^2}{N_c} \langle \textcolor{red}{E}_i^a(\vec{b}_1) \textcolor{red}{E}_j^b(\vec{b}_2) \rangle = \frac{1}{N_c^2 - 1} \delta^{ab} \delta_{ij} Q_s^2 \Delta(\vec{b}_1 - \vec{b}_2)$$

- B The above averages over all fluctuations of target fields, and hence is isotropic. For observables which are sensitive to angular structure of target fields, instead integration over target field ensembles subject to constraint that anisotropic contribution to electric field point in specific direction \hat{a} should be performed

$$\frac{g^2}{N_c} \langle \textcolor{red}{E}_i^a(\vec{b}_1) \textcolor{red}{E}_j^b(\vec{b}_2) \rangle_{\hat{a}} = \frac{1}{N_c^2 - 1} \delta^{ab} Q_s^2 \Delta(\vec{b}_1 - \vec{b}_2) \left(\delta_{ij} + 2\mathcal{A} \left[\hat{a}_i \hat{a}_j - \frac{1}{2} \delta_{ij} \right] \right)$$

A. Dumitru, L. McLerran, V. S. 1410.4844

CUMULANTS OF AZIMUTHAL ANISOTROPY I

- Conventional $\langle \vec{E}(\vec{b}_1) \vec{E}(\vec{b}_2) \rangle$

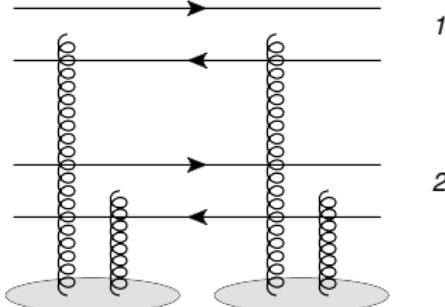
$$\frac{g^2}{N_c} \langle \textcolor{red}{E}_i^a(\vec{b}_1) \textcolor{red}{E}_j^b(\vec{b}_2) \rangle = \frac{1}{N_c^2 - 1} \delta^{ab} \delta_{ij} Q_s^2 \Delta(\vec{b}_1 - \vec{b}_2)$$

- Cumulants of azimuthal anisotropy can be readily computed. Cumulants are defined in such a way as to cancel disconnected pieces not associated with **single** particle azimuthal anisotropy. For example

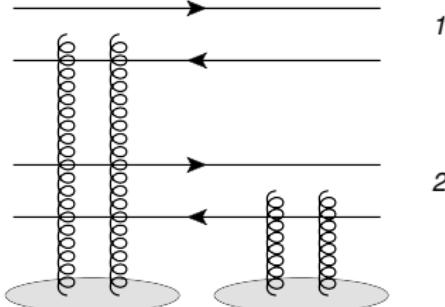
$$c_n\{4\} = \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle_{\text{conn}} = \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle - 2 \langle e^{in(\phi_1 - \phi_3)} \rangle^2$$

- In field theory, this corresponds to considering fully connected diagrams only

Connected graph



Disconnected graph



CUMULANTS OF AZIMUTHAL ANISOTROPY II

- There are $(2m - 2)!!$ ways to contract $S_m(\vec{r}_1, \vec{b}_1, \dots, \vec{r}_m, \vec{b}_m)$ in fully connected way:

$$\langle S_m(\vec{r}_1, \vec{b}_1, \dots, \vec{r}_m, \vec{b}_m) - 1 \rangle^{\text{conn.}} = \left(\frac{-Q_s^2}{4} \right)^m \frac{1}{(N_c^2 - 1)^{m-1}}$$
$$\Delta(\vec{b}_1 - \vec{b}_2)\Delta(\vec{b}_2 - \vec{b}_1) \cdots \Delta(\vec{b}_{m-1} - \vec{b}_m)\Delta(\vec{b}_m - \vec{b}_1)$$
$$(\vec{r}_1 \vec{r}_2)(\vec{r}_2 \vec{r}_3) \cdots (\vec{r}_{m-1} \vec{r}_m)(\vec{r}_m \vec{r}_1) + \text{permutations.}$$

- Averaging with respect to impact parameters (for Gaussian $\Delta(\vec{b})$):

$$\langle S_m(\vec{r}_1, \dots, \vec{r}_m) - 1 \rangle^{\text{conn.}} = \left(\frac{-Q_s^2}{4} \right)^m \frac{1}{(N_c^2 - 1)^{m-1}} \frac{\xi^{m-1}}{m}$$
$$(\vec{r}_1 \vec{r}_2)(\vec{r}_2 \vec{r}_3) \cdots (\vec{r}_{m-1} \vec{r}_m)(\vec{r}_m \vec{r}_1) + \text{permutations.}$$

$$\xi = S_c / S_p = 1 / N_D.$$

- In large N_c , normalization of angular averages is defined by disconnected contribution

$$\langle S_m(\vec{r}_1, \dots, \vec{r}_m) - 1 \rangle^{\text{disc.}} \approx \left(-\frac{Q_s^2}{4} \right)^m \prod_{i=1}^m r_i^2.$$

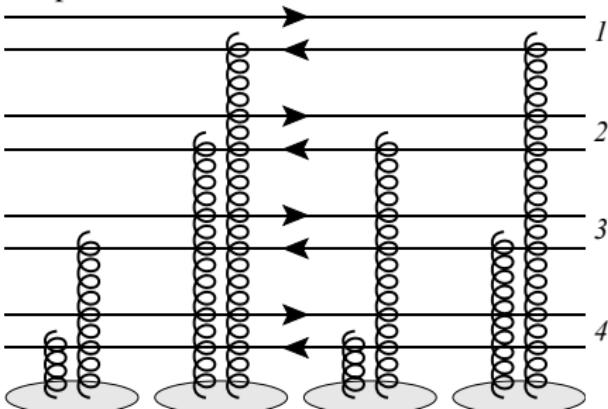
CUMULANTS OF AZIMUTHAL ANISOTROPY III

- Not all $(2m - 2)!!$ terms contribute to cumulants. $m!!(m - 2)!!$ nonzero terms are defined by all possible contractions of terms entering with opposite signs before ϕ 's in $e^{2i(\phi_1 + \phi_2 + \dots + \phi_n - \phi_{n+1} - \phi_{n+2} - \dots - \phi_{2n})}$. E.g.

$$\propto (\vec{r}_1 \vec{r}_{n+1})(\vec{r}_1 \vec{r}_{n+2})(\vec{r}_2 \vec{r}_{n+2})(\vec{r}_2 \vec{r}_{n+3}) \cdots (\vec{r}_{n-1} \vec{r}_{2n-1})(\vec{r}_{n-1} \vec{r}_{2n})(\vec{r}_n \vec{r}_{2n})(\vec{r}_n \vec{r}_{n+1})$$

- For 4 particles, $e^{2i(\phi_1 + \phi_2 - \phi_3 - \phi_4)}$

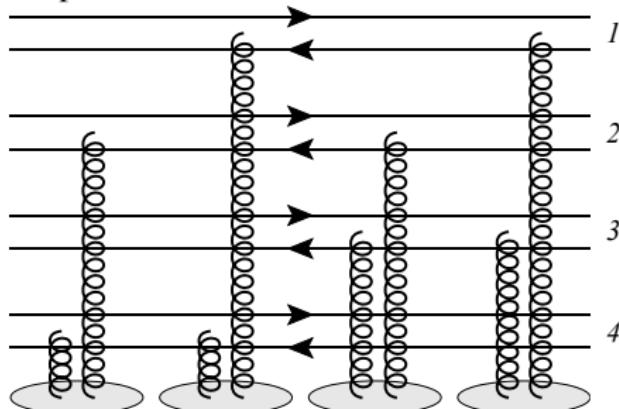
Graph that does not contribute



$$\propto (\vec{r}_1 \vec{r}_3)(\vec{r}_2 \vec{r}_4)(\vec{r}_1 \vec{r}_2)(\vec{r}_3 \vec{r}_4)$$

V. S. 1412.5191

Graph that does contribute



$$\propto (\vec{r}_1 \vec{r}_3)(\vec{r}_1 \vec{r}_4)(\vec{r}_2 \vec{r}_3)(\vec{r}_2 \vec{r}_4)$$

$c_2\{m\}$ FROM CONNECTED DIAGRAMS

- Final result

$$c_2\{m\} = \frac{m!!(m-2)!!}{m \cdot 2^m} \left(\frac{\xi}{N_c^2 - 1} \right)^{m-1}$$

- Suppressed by powers of $1/N_c^2$ and $\xi = S_c/S_p$.
- $c_2\{m\}$ are manifestly positive for any m .
- Same result remains true for adjoint representation (Casimir operators cancel in normalized observables).

$v_2\{m\}$ FROM CONNECTED DIAGRAMS I

- Harmonics are related to cumulants. Relation of flow coefficients to cumulants
(see N. Borghini, P. M. Dinh and J. Y. Ollitrault 0105040)

$$v_2^{2k}\{2k\} = (-1)^{k+1} \times (\text{Numerical coefficient}) \times c_2\{m = 2k\} = \kappa_{2k} c_2\{m = 2k\}$$

- First few κ_{2m} :

Order, $2m$	2	4	6	8	10	12	14
$1/\kappa_{2m}$	1	-1	-4	33	-456	9460	-274800

- Idea behind these numbers: if we have dominating **single** particle azimuthal anisotropy $v_2\{1\}$ then

$$v_2^m\{m\} = v_2^m\{1\} + \text{corrections}$$

Purpose for hydrodynamics: extract genuine $v_2\{1\}$ and suppress “non-flow”.

$v_2\{m\}$ FROM CONNECTED DIAGRAMS II

- To derive κ_{2m} we need to compute coefficient before $\langle Q^{2m} \rangle$ in expansion of generating function $\mathcal{G} = \langle \ln(I_0(2xQ)) \rangle$ at $x = 0$ and divide it by proper normalization

$$\mathcal{N}_{2m} = \left. \frac{d^{2m}}{dx^{2m}} \ln(I_0(2x)) \right|_{x=0}.$$

- Coefficient before $\langle Q^{2m} \rangle$ can be computed by performing expansion of Bessel function

$$\frac{(2m)!}{(m!)^2}$$

For normalization \mathcal{N}_{2m} use $I_0(2x) = \prod_{k=1}^{\infty} \left(1 + \left(\frac{2x}{j_{0,k}}\right)^2\right)$ to get

$$\begin{aligned}\mathcal{N}_{2m} &= \sum_{k=1}^{\infty} \left. \frac{d^{2m}}{dx^{2m}} \ln\left(1 + \left(\frac{2x}{j_{0,k}}\right)^2\right) \right|_{x=0} \\ &= \sum_{k=1}^{\infty} \left. \frac{d^{2m}}{dx^{2m}} \left[\sum_i \frac{(-1)^{i+1}}{i} \left(\frac{2x}{j_{0,k}}\right)^{2i} \right] \right|_{x=0} = (-1)^{m+1} \frac{(2m)!}{m} \sum_{k=1}^{\infty} \left(\frac{2}{j_{0,k}}\right)^{2m}.\end{aligned}$$

Finally

$$\kappa_{2m} = \frac{v_n^{2m}\{2m\}}{c_n\{2m\}} = \frac{(2m)!}{(m!)^2 \mathcal{N}_{2m}} = (-1)^{m+1} \left[m!(m-1)! \sum_{k=1}^{\infty} \left(\frac{2}{j_{0,k}}\right)^{2m} \right]^{-1}.$$

$v_2\{m\}$ FROM CONNECTED DIAGRAMS III

- Harmonics:

$$(v_2\{m\})^m = \frac{(-1)^{\frac{m}{2}+1}}{m\beta_m} \left(\frac{\xi}{N_c^2 - 1} \right)^{m-1}; \quad \beta_m = 2 \sum_{k=1}^{\infty} \left(\frac{2}{j_{0,k}} \right)^m \approx 2 \left(\frac{2}{j_{0,1}} \right)^m$$

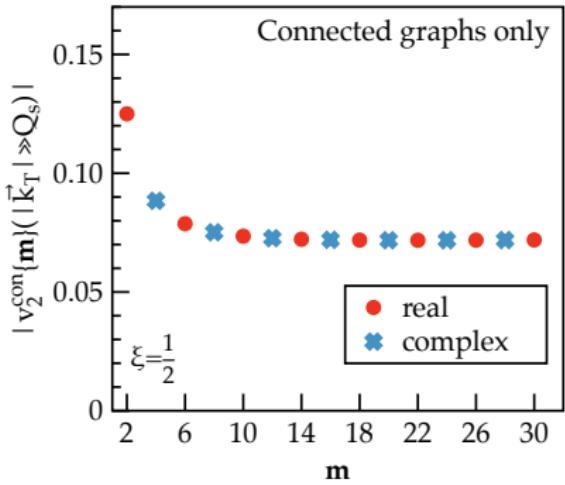
- Explicitly for second and fourth order:

$$v_2^2\{2\} = \frac{1}{4} \frac{\xi}{N_c^2 - 1}; \quad v_2^4\{4\} = -\frac{1}{4} \left(\frac{\xi}{N_c^2 - 1} \right)^3$$

- $v_2\{2\}$ has no anomalies
- $v_2\{4\}$ is complex!
- $m \rightarrow \infty$:

$$\lim_{m \rightarrow \infty} |v_2\{m\}| = \frac{\xi}{N_c^2 - 1} \frac{j_{0,1}}{2}; \quad j_{0,1} = 2.40483$$

$v_2\{m\}$ FROM CONNECTED DIAGRAMS: ILLUSTRATION



A. Dumitru, L. McLerran, V. S. 1410.4844

V. S. 1412.5191

- A hydro practitioner: “non-flow”. However, very different from conventional non-flow contributions (i.e. resonance decay): long-range in rapidity, approximate equality of high order harmonics $|v_2\{m\}|$.

- Hierarchy of $|v_2\{m\}|$.
- **Complex $v_2\{4k\}, k \in \mathbb{Z}$; including $v_2\{4\}$ and $v_2\{8\}$**
- Experiment: high multiplicity pA $c_2\{4\} < 0 \sim v_2\{4\} \in \mathcal{R}$
- **Theory: connected graph only** $c_2\{4\} > 0 \sim v_2\{4\} \in \mathcal{C}$
- In order to describe high k_\perp with IS effects, one needs disconnected graphs with azimuthal anisotropy
- I believe that similar conclusion is valid for dense-dense limit and “glasma” graph

SINGLE PARTICLE ANISOTROPY

- Solution: single particle azimuthal anisotropy
- Kovner&Lublinsky: $\text{Re } S_\rho - 1 = \frac{(ig)^2}{2N_c} \text{tr}(\vec{r}\vec{E})^2$; If projectile partons scatter off same electric field, they pick up same transverse momentum (independent on rapidity of partons).
- To take this into account, let modify $E - E$ correlator:

$$\frac{g^2}{2N_c} \langle \text{tr } E_i(\vec{b}_{1\perp}) E_j(\vec{b}_{2\perp}) \rangle = \frac{1}{4} Q_s^2 \Delta(\vec{b}_{1\perp} - \vec{b}_{2\perp}) \left(\delta^{ij} + 2 \mathcal{A} \left(\hat{a}^i \hat{a}^j - \frac{1}{2} \delta^{ij} \right) \right)$$

- Angular distribution for scattering of single dipole, for fixed \hat{a} :

$$\left(\frac{1}{\pi} \frac{dN}{dk^2} \right)^{-1} \frac{dN}{d^2 k} = 1 - 2\mathcal{A} + 4\mathcal{A}(\hat{k} \cdot \hat{a})^2.$$

Consequently, elliptic harmonic of single-particle distribution: $v_2 \equiv \langle e^{2i(\phi_k - \phi_a)} \rangle_{\hat{a}} = \mathcal{A}$.

- Repeating calculations, see details in [†]:

$$v_2^2\{2\} = c_2\{2\} = \xi \left(\mathcal{A}^2 + \frac{1}{4(N_c^2 - 1)} \right)$$

$$v_2^4\{4\} = -c_2\{4\} = \xi^3 \left(\mathcal{A}^4 - \frac{1}{4(N_c^2 - 1)^3} \right)$$

Factors of $\xi = 1/N_D$: partons scatter off domain with same \vec{E} orientation.

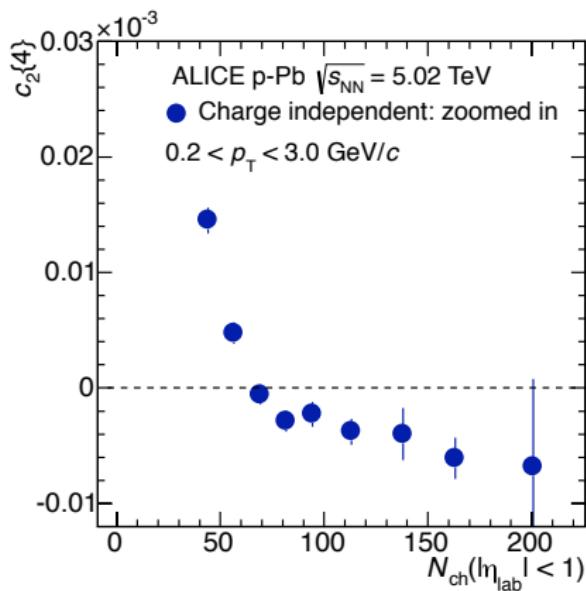
A. Kovner, M. Lublinsky, 1109.0347, 1211.1928;

[†] A. Dumitru, L. McLerran, V.S. 1410.4844;

A. Dumitru, A. Giannini 1406.5781

A. Dumitru, V. S. 1411.6630

SINGLE PARTICLE ANISOTROPY



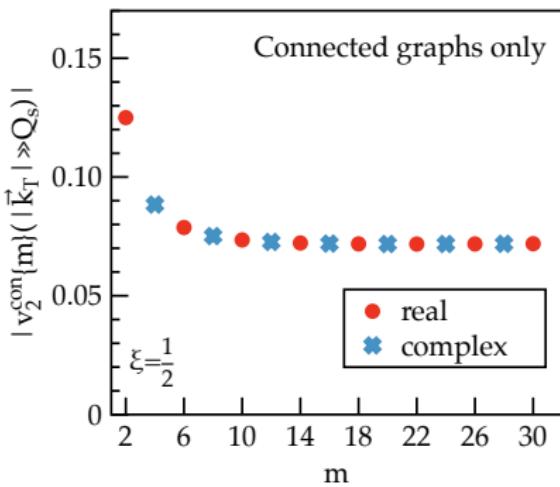
- $c_2\{4\} = -\xi^3 \left(\mathcal{A}^4 - \frac{1}{4(N_c^2-1)^3} \right)$
- Interpretation in terms of IS:
 - $N_{ch} < 60$: dominated by **connected** contribution to $c_2\{4\}$
 - $N_{ch} > 60$: dominated by single particle **disconnected** contribution to $c_2\{4\}$

ALICE Coll. 1406.2474

SINGLE PARTICLE ANISOTROPY: ILLUSTRATION OF HIERARCHY

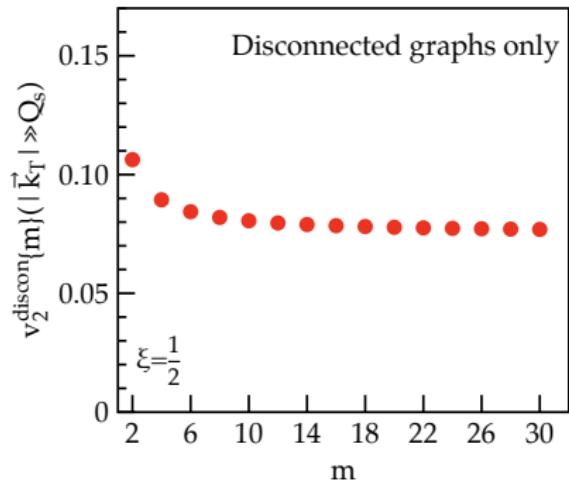
- In general it is immensely difficult to derive general expression for $v_2\{m\}$.
- Limiting cases are easy:
- Connected graphs dominate:

$$(v_2\{m\})^m = \frac{(-1)^{\frac{m}{2}+1}}{m\beta_m} \left(\frac{\xi}{N_c^2 - 1} \right)^{m-1};$$



- \mathcal{A} (single particle anisotropy) dominates:

$$v_2\{m\} = \xi^{1-1/m} \mathcal{A}; \quad \text{at large } m \quad v_2\{m\} = \xi \mathcal{A}$$



MV MODEL FOR HIGH ENERGY

- Large-x valence partons are modelled by random, recoilless color charges $\rho^a(\vec{x}_\perp)$ creating semi-classical small-x gluon fields $A^a(\vec{x}_\perp)$.
- Gaussian distribution of sources

$$S_{\text{eff}}[\rho^a] = \int dx^- d^2x_\perp \frac{\rho^a(x^-, \vec{x}_\perp) \rho^a(x^-, \vec{x}_\perp)}{2\mu^2}$$

- Weizsäcker-Williams fields:

$$A^{\mu a}(x^-, \vec{x}_\perp) = -\delta^{\mu+} \frac{g}{\nabla_\perp^2} \rho^a(x^-, \vec{x}_\perp).$$

- Propagation of fundamental charge in this field

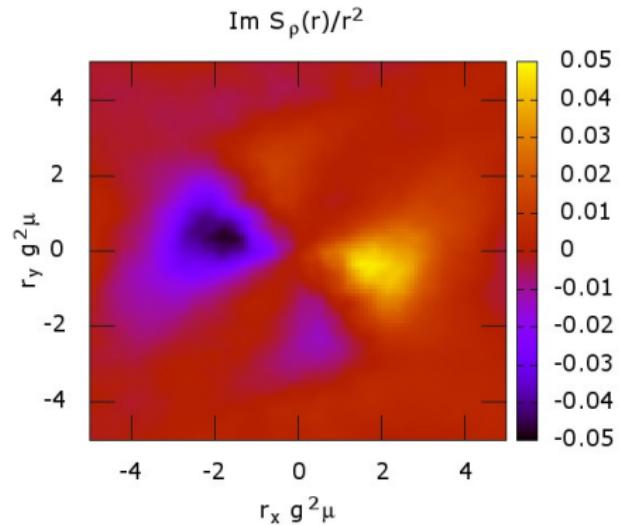
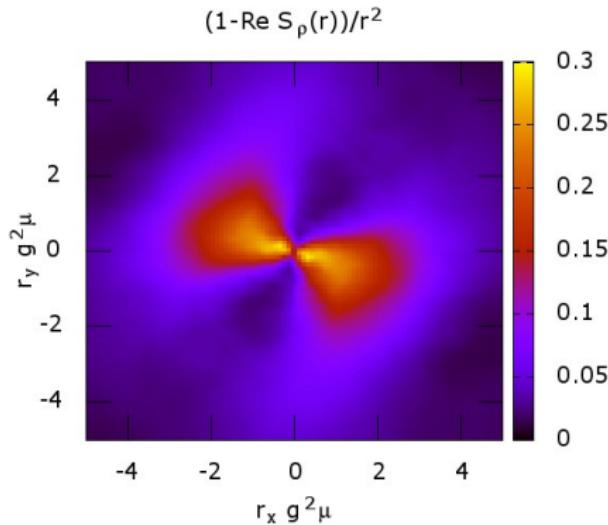
$$V(\vec{x}_\perp) = \mathbb{P} \exp \left\{ -ig \int dx^- t^a A^{+a}(x^-, \vec{x}_\perp) \right\}$$

- S-matrix for scattering charge off given target field configuration

$$S_\rho(\vec{r}_\perp, \vec{b}_\perp) \equiv \frac{1}{N_c} \text{tr} V^\dagger(\vec{x}_\perp) V(\vec{y}_\perp), \quad \vec{r}_\perp \equiv \vec{x}_\perp - \vec{y}_\perp, \quad 2\vec{b}_\perp \equiv \vec{x}_\perp + \vec{y}_\perp$$

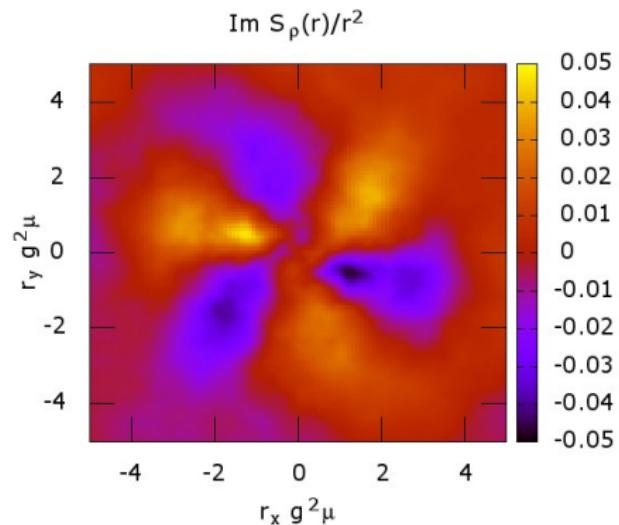
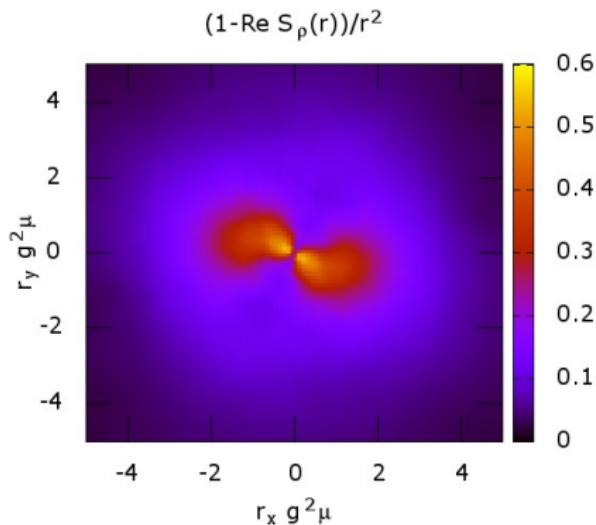
Details on numerical implementation:
A. Dumitru, V. S. 1411.6630
T. Lappi 0711.3039

SINGLE CONFIGURATION I



- Normalization $1/r^2$ mimics LO of isotropic part of cross-section
- Real part is dominated by $\cos 2\phi_r$
- Imaginary part is dominated by $\cos \phi_r$

SINGLE CONFIGURATION II



- Higher orders are seen in real part
- Imaginary part is dominated by $\cos 3\phi_r$

A. Dumitru, V. S. 1411.6630

AMPLITUDES: DEFINITION

- Goal is to extract amplitudes of azimuthal anisotropy
- In each event Fourier decomposition is performed:

$$1 - \operatorname{Re} S_\rho(\vec{r}_\perp) \equiv D_\rho(\vec{r}_\perp) = \mathcal{N}(r_\perp) \left(1 + \sum_{n=1}^{\infty} \textcolor{red}{A'_{2n}(r_\perp)} \cos(2n\phi_r) \right)$$

$$\operatorname{Im} S_\rho(\vec{r}_\perp) = \mathcal{N}(r_\perp) \sum_{n=0}^{\infty} \textcolor{red}{A'_{2n+1}(r_\perp)} \cos((2n+1)\phi_r)$$

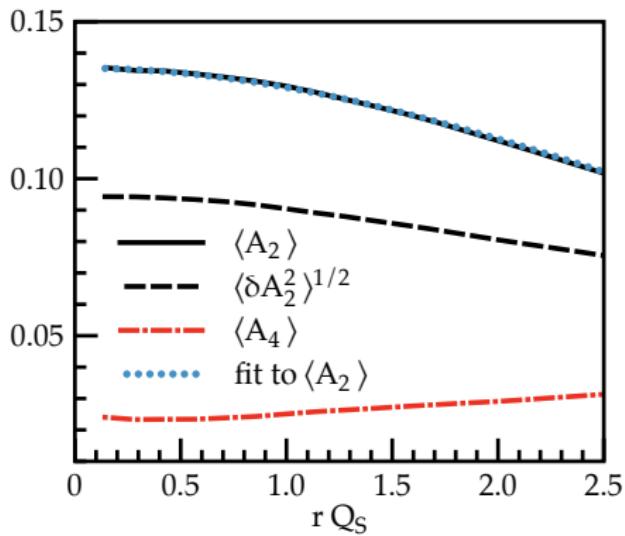
- To cancel trivial random phase, which corresponds to global rotation of charge distribution ρ from configuration to configuration

$$\textcolor{red}{A_n(r_\perp)} = |A'_{2n}(r_\perp)|$$

- Results are presented in terms of Q_s defined by $\langle S_\rho \rangle (r_s = \sqrt{2}/Q_s) \stackrel{!}{=} e^{-1/2}$

A. Dumitru, V. S. 1411.6630

AMPLITUDES: EVEN HARMONICS



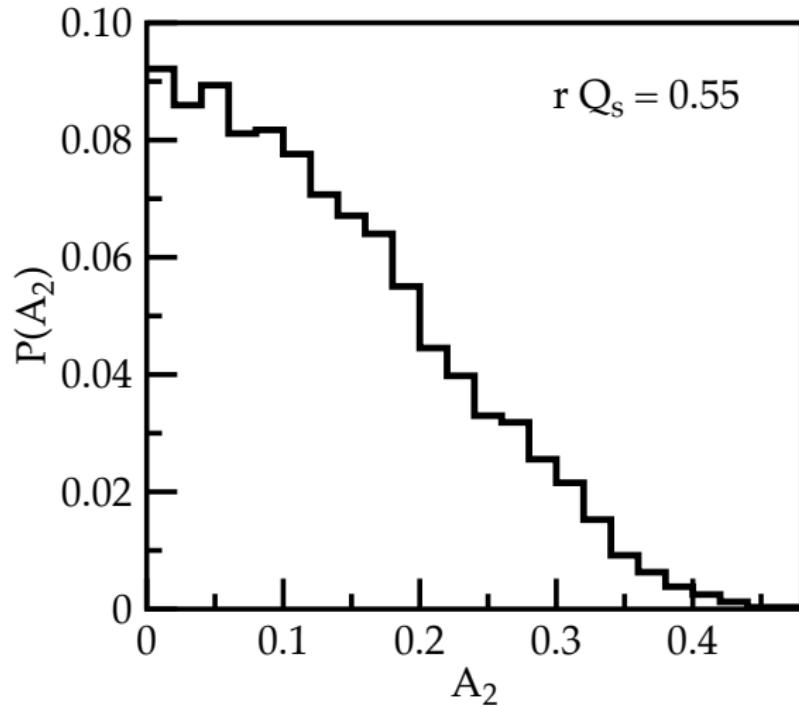
- Largest amplitude: $\langle A_2 \rangle$
- Finite $\langle A_2 \rangle$ at $r \rightarrow 0$; $\langle A_2 \rangle$ is approximately constant for $r < 1/Q_s$
- $\langle \delta A_2^2 \rangle^{1/2}$ is comparable to $\langle A_2 \rangle$: fluctuations are rather high
- $\langle A_4 \rangle$ is significantly smaller
- Fit of $\langle A_2 \rangle$ motivated by h_1^g of distribution of linearly polarized gluons (for an unpolarized target) introduced in TMD factorization

$$\delta^{ij} f_1^g(x, \vec{k}^2) + \left(\hat{k}^i \hat{k}^j - \frac{1}{2} \delta^{ij} \right) h_1^{\perp g}(x, \vec{k}^2).$$

In MV (A. Metz and J. Zhou, 1105.1991):

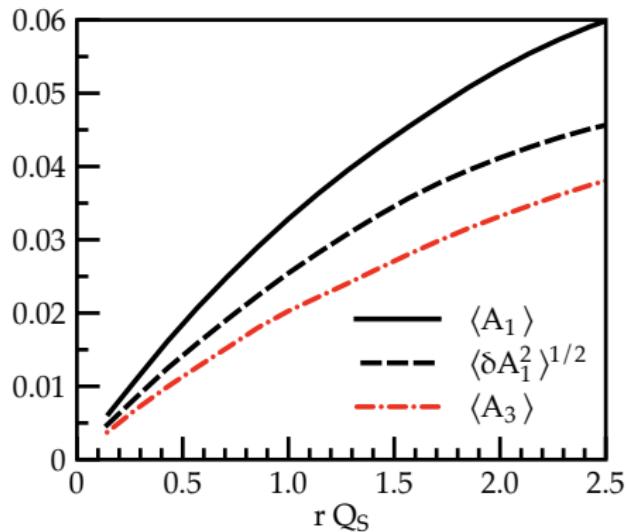
$$h_1^{\perp g}(x, \vec{r}^2) \propto \frac{1}{r^2 Q_s^2} \left[1 - \exp \left(-\frac{r^2 Q_s^2}{4} \right) \right]$$

FLUCTUATIONS OF A_2



A. Dumitru, V. S. 1411.6630

AMPLITUDES: ODD HARMONICS



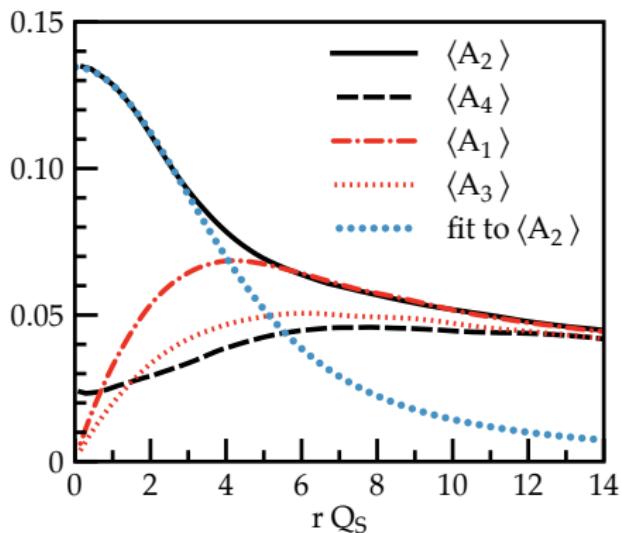
- Expectation value of $\text{Im } S_\rho$ is 0
- However, odd harmonics are non-zero!
- At small r , $\langle A_1 \rangle$ and $\langle A_3 \rangle$ approach zero, as expected from analytic arguments

(A. Dumitru and A. Giannini, 1406.5781)

$$\text{Im } S_\rho \propto \alpha_s r^3 \cos \phi_r$$

A. Dumitru, V. S. 1411.6630

AMPLITUDES: LARGE r

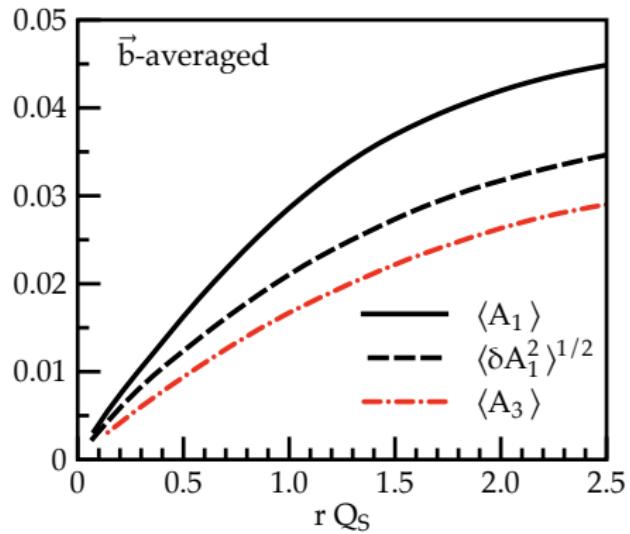
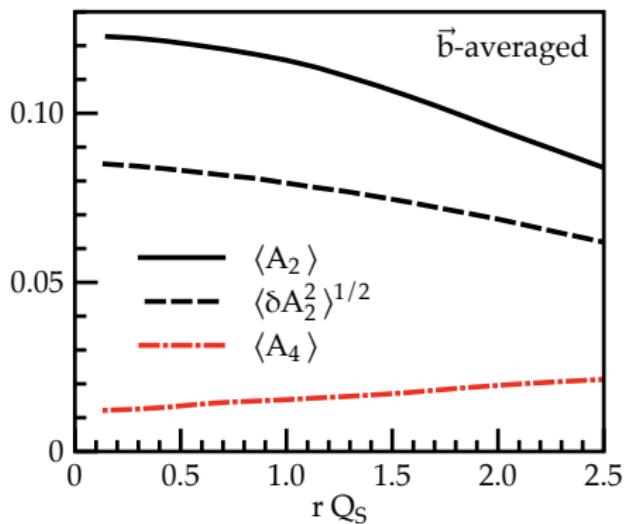


- Fit breaks down at large $r > 3Q_s$: analytical derivation involves adhoc IR cut-offs introduced arbitrary
- Amplitudes approach common non-zero function at large r : expected universal scale invariance of fluctuations of azimuthal dependence of S-matrix

A. Dumitru, V. S. 1411.6630

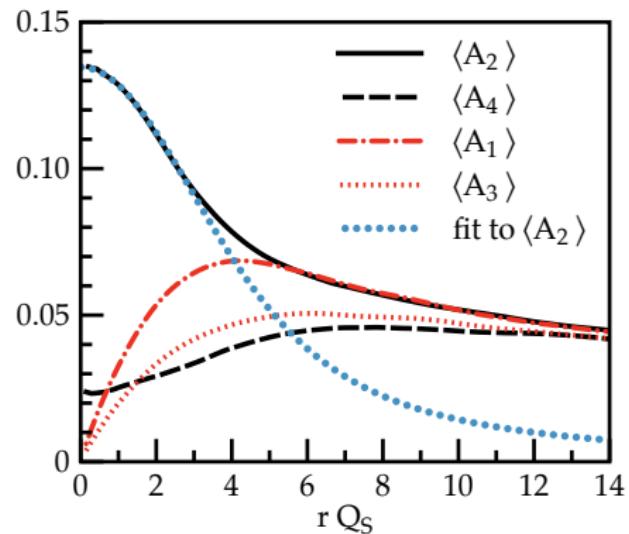
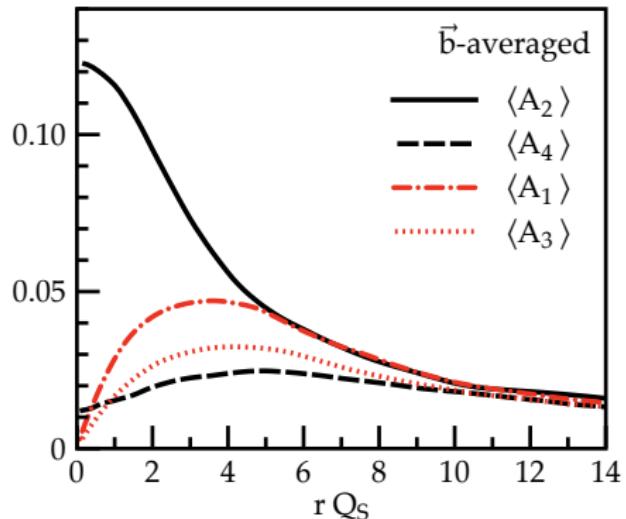
AMPLITUDES: AVERAGING OVER FINITE \vec{b}_\perp

$$\overline{D}_\rho(\vec{r}_\perp, \vec{b}_\perp) = \int \frac{d^2 \vec{b}'_\perp}{\pi r_\perp^2} \Theta(r_\perp - |\vec{b}_\perp - \vec{b}'_\perp|) D_\rho(\vec{r}_\perp, \vec{b}_\perp)$$



A. Dumitru, V. S. 1411.6630

AMPLITUDES: AVERAGING OVER FINITE \vec{b}_\perp



A. Dumitru, V. S. 1411.6630

BEYOND MV

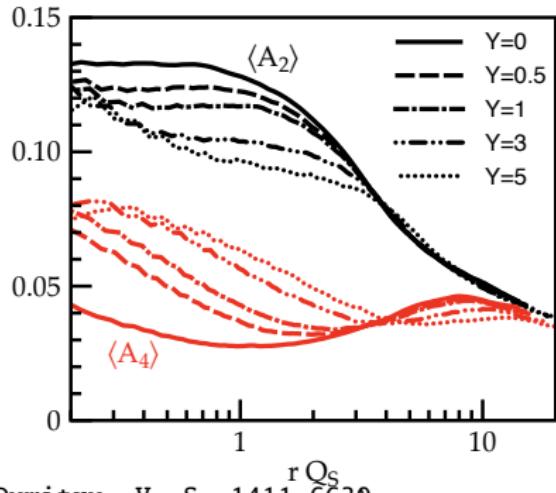
- Beyond MV: inclusion of quantum fluctuations.
- Previous “**mean-field**” **BK studies**
 - showed that azimuthal anisotropy decreases exponentially in Y .
See A. Kovner & M. Lublinsky 1211.1928.
 - Anisotropic initial conditions $1 - S \propto e^{-1/4Q_S^2 r^2(1+\# \cos(2\phi))}$
 - BK equation assuming uniform distribution in impact parameter space
- $$\partial_Y N(\vec{r}) = \frac{C_F \alpha(r^2)}{2\pi} \int d^2 \vec{r}_1 K(\vec{r}, \vec{r}_1) (N(\vec{r}_1) + N(\vec{r}_2) - N(\vec{r}) - N(\vec{r}_1)N(\vec{r}_2))$$
- In this particular set up I was able to reproduce their conclusion: azimuthal anisotropy decays very fast with $Y = \ln(x_0/x)$.
 - **Crucial assumption:** uniformity in impact parameter space
 - Even at level of initial condition (given by MV model), azimuthal anisotropy of $S(\vec{r}, \vec{b})$ arises due to fluctuations of soft fields in transverse impact parameter plane.
 - **JIMWLK** equation with both \vec{r} and \vec{b} .

JIMWLK I

Evolution over a step ΔY in rapidity opens up phase space for radiation of gluons and modifies classical action. This is taken into account by functional renormalization group equation, JIMWLK.

In terms of ‘random walk’ in space of Wilson lines
(see e.g. T. Lappi and H. Mäntysaari, 1212.4825):

$$\partial_Y V(\vec{x}) = V(\vec{x}) \frac{i}{\pi} \int d^2 \vec{u} \frac{(\vec{x} - \vec{u})^i \eta^i(\vec{u})}{(\vec{x} - \vec{u})^2} - \frac{i}{\pi} \int d^2 \vec{v} V(\vec{v}) \frac{(\vec{x} - \vec{v})^i \eta^i(\vec{v})}{(\vec{x} - \vec{v})^2} V^\dagger(\vec{v}) V(\vec{x}).$$



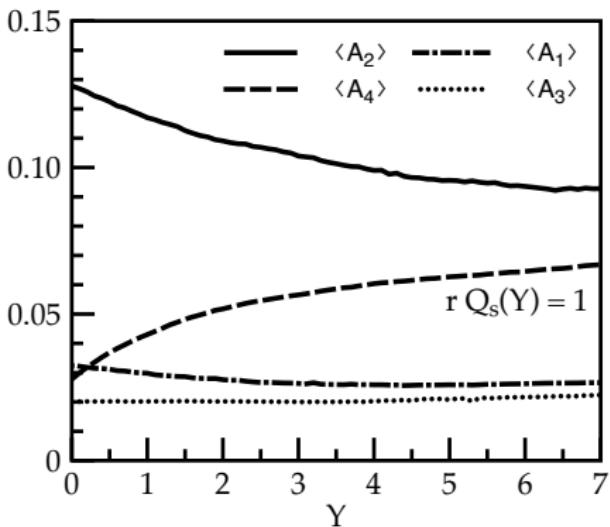
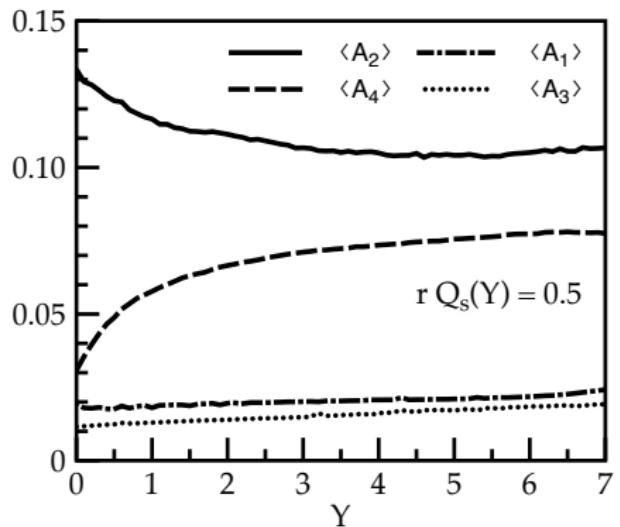
A. Dumitru, V. S. 1411.6630

VLADIMIR.SKOKOV@WMICH.EDU

AZIMUTHAL ANISOTROPY

BNL 2015 34 / 37

JIMWLK II



A. Dumitru, V. S. 1411.6630

ODD HARMONICS

- Single particle anisotropy:
 - [A] for fundamental representation S_F has odd harmonics, e.g. $\cos(3\phi)$;
 - [B] for adjoint representation S_A is manifestly real and thus can have only even harmonics (S_A)
- Two particle azimuthal anisotropy: [A] does not help much if there is an approximate quark—anti-quark symmetry of projectile wave function at small x . Indeed, two particle correlation function summed over qq , $q\bar{q}$, $\bar{q}q$ and $\bar{q}\bar{q}$ channels is C -even

$$S_2 \propto \left(\text{tr } V^\dagger(\vec{x}_1) V(\vec{y}_1) + \text{tr } V(\vec{x}_1) V^\dagger(\vec{y}_1) \right) \left(\text{tr } V^\dagger(\vec{x}_2) V(\vec{y}_2) + \text{tr } V(\vec{x}_2) V^\dagger(\vec{y}_2) \right)$$

is real, and so has even cumulants only.

- Obtaining non-zero $c_1\{2\}$ and $c_3\{2\}$ may require to account for (at least) one additional soft rescattering of (anti-) quarks besides their hard scattering from target shockwave.

CONCLUSIONS

- Neglecting initial state one-particle azimuthal anisotropy: complex $v_2\{4\}$
- Initial state one-particle azimuthal anisotropy is present due to fluctuating valence quarks
- Disconnected contribution to $c_2\{4\}$ results in real $v_2\{4\}$
- MV model:

even amplitudes $\langle A_2 \rangle$ and $\langle A_4 \rangle$ of azimuthal anisotropy are approximately constant for $k_\perp > Q_s$ ($r_\perp < 1/Q_s$);

odd amplitudes $\langle A_1 \rangle$ and $\langle A_3 \rangle$ approach zero at $k_\perp \rightarrow \infty$ ($r_\perp \rightarrow 0$)

$$\langle A_2(k_\perp \sim Q_s) \rangle \approx 13\%$$

$$\langle A_1(k_\perp \sim Q_s) \rangle \approx 3\%$$

- Small- x evolution does not significantly alter anisotropy
- Does $v_2\{m\}$ has a direct connection to TMD h_1^g ??!
- No multiplicity bias were imposed in our studies.